UNIVERZITET U ISTOČNOM SARAJEVU
FAKULTET ZA PROIZVODNJU I MENADŽMENT TREBINJE
Predmet: Mehanika fluida

RIJEŠENI ZADACI IZ STATIKE FLUIDA

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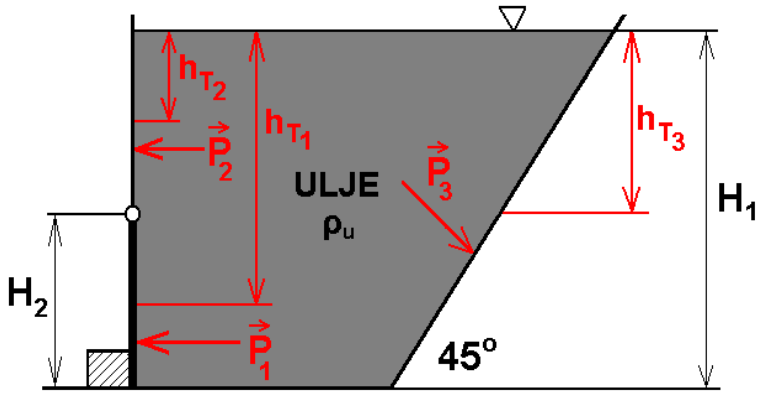
Trebinje, novembar 2014. godine

1. Zadatak

U rezervoaru širine B = 1 m prikazanom na skici nalazi se ulje (ρ_u = 1200 kg/m^3) do visine H_1 = 2 m. Odrediti:

a) silu koja djeluje na zatvarač visine H_2 = 1 m i širine B = 1 m i silu koja djeluje na zid iznad zatvarača;

b) silu kojom ulje djeluje na zid koji je pod uglom od 45° u odnosu na horizontalu.



Rješenje:

a)

$$P_1 = \rho_u \cdot g \cdot h_{T_1} \cdot A_1,$$

$$h_{T_1} = H_1 - \frac{H_2}{2} = 2 - \frac{1}{2} = 1{,}5 \ \ m,$$

$$A_1 = H_2 \cdot B = 1 \cdot 1 = 1 \ \ m^2,$$

$$P_1 = 1200 \cdot 9{,}81 \cdot 1{,}5 \cdot 1,$$

$$P_1 = 17658 \ \ N.$$

$$P_2 = \rho_u \cdot g \cdot h_{T_2} \cdot A_2,$$

$$h_{T_2} = \frac{H_1 - H_2}{2} = \frac{2-1}{2} = 0{,}5 \ \ m,$$

$$A_2 = (H_1 - H_2) \cdot B = (2-1) \cdot 1 = 1 \ \ m^2,$$

$$P_2 = 1200 \cdot 9{,}81 \cdot 0{,}5 \cdot 1,$$

$$P_2 = 5886 \ \ N.$$

b)

$$P_3 = \rho_u \cdot g \cdot h_{T_3} \cdot A_3,$$

$$h_{T_3} = \frac{H_1}{2} = \frac{2}{2} = 1 \ \ m,$$

$$A_3 = \frac{H_1}{\sin 45^o} \cdot B = \frac{2}{\sqrt{2}/2} \cdot 1 = \frac{4}{\sqrt{2}} = 2\sqrt{2} \ \ m^2,$$

$$P_3 = 1200 \cdot 9{,}81 \cdot 1 \cdot 2\sqrt{2},$$

$$P_3 = 33197{,}04 \ \ N.$$

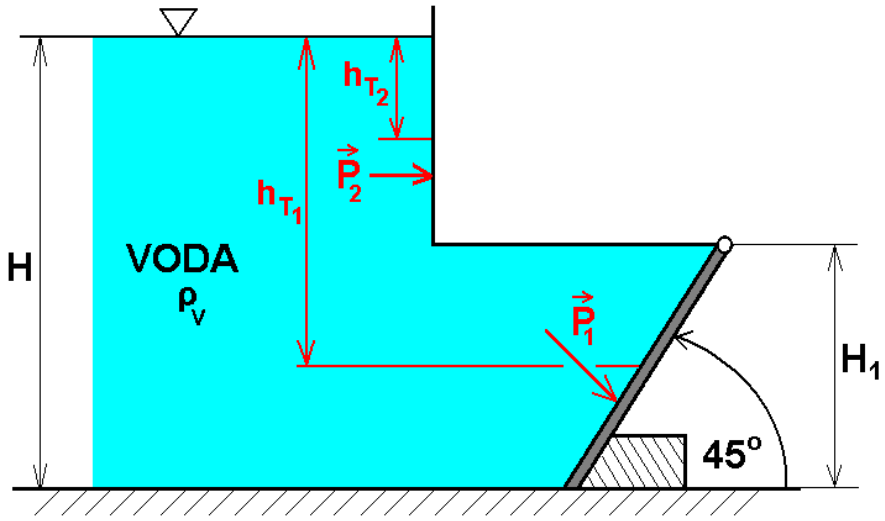
2. Zadatak

U rezervoaru, širine $B = \sqrt{2}\ m$, nalazi se voda (ρ_v = 1000 kg/m^3) do visine H = 2 m.

Odrediti:

a) silu koja djeluje na zatvarač (širine $B = \sqrt{2}\ m$) koji je pod uglom od 45° u odnosu na horizontalu, ako je H_1 = 1,1 m.

b) silu koja djeluje na vertikalni zid iznad zatvarača.



Rješenje:

a)

$$P_1 = \rho_v \cdot g \cdot h_{T_1} \cdot A_1,$$

$$h_{T_1} = H - \frac{H_1}{2} = 2 - \frac{1{,}1}{2} = 1{,}45\ \ m,$$

$$A_1 = \frac{H_1}{\sin 45^o} \cdot B = \frac{1{,}1}{\sqrt{2}/2} \cdot \sqrt{2} = \frac{2 \cdot 1{,}1}{\sqrt{2}} \cdot \sqrt{2} = 2{,}2\ \ m^2,$$

$$P_1 = 1000 \cdot 9{,}81 \cdot 1{,}45 \cdot 2{,}2,$$

$$P_1 = 31293{,}9\ \ N.$$

b)

$$P_2 = \rho_v \cdot g \cdot h_{T_2} \cdot A_2,$$

$$h_{T_2} = \frac{H - H_1}{2} = \frac{2 - 1{,}1}{2} = 0{,}45\ \ m,$$

$$A_2 = (H - H_1) \cdot B = (2 - 1{,}1) \cdot \sqrt{2} = 1{,}27\ \ m^2,$$

$$P_2 = 1000 \cdot 9{,}81 \cdot 0{,}45 \cdot 1{,}27,$$

$$P_2 = 5606{,}42\ \ N.$$

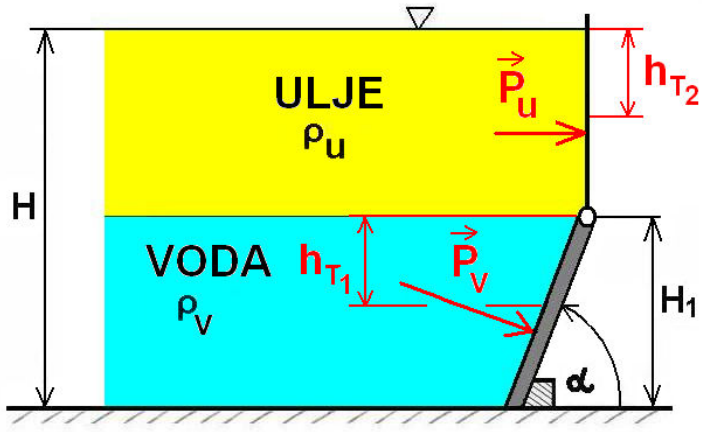
3. Zadatak

U rezervoaru, širine $B = \sqrt{3}\ m$, nalazi se voda (ρ_v = 1000 kg/m^3) do visine H_1 = 1 m, a iznad nje ulje (ρ_u= 800 kg/m^3) do visine H = 2 m.

Odrediti:

a) silu koja djeluje na zatvarač (širine $B=\sqrt{3}\ m$) ako je α = 60 ^{o};

b) silu koja djeluje na zid iznad zatvarača.



Rješenje:

a)

$$P_v = \rho_v \cdot g \cdot h_{T_1} \cdot A_1 + \rho_u \cdot g \cdot (H - H_1) \cdot A_1,$$

$$h_{T_1} = \frac{H_1}{2} = \frac{1}{2}\ \ m,$$

$$A_1 = \frac{H_1}{\sin\alpha} \cdot B = \frac{1}{\sqrt{3}/2} \cdot \sqrt{3} = \frac{2}{\sqrt{3}} \cdot \sqrt{3} = 2\ \ m^2,$$

$$P_v = 1000 \cdot 9{,}81 \cdot \frac{1}{2} \cdot 2 + 800 \cdot 9{,}81 \cdot (2-1) \cdot 2,$$

$$P_v = 9810 + 15696,$$

$$P_v = 25506\ \ N.$$

b)

$$P_u = \rho_u \cdot g \cdot h_{T_2} \cdot A_2,$$

$$h_{T_2} = \frac{H - H_1}{2} = \frac{2-1}{2} = \frac{1}{2}\ \ m,$$

$$A_2 = (H - H_1) \cdot B = (2-1) \cdot \sqrt{3} = \sqrt{3}\ \ m^2,$$

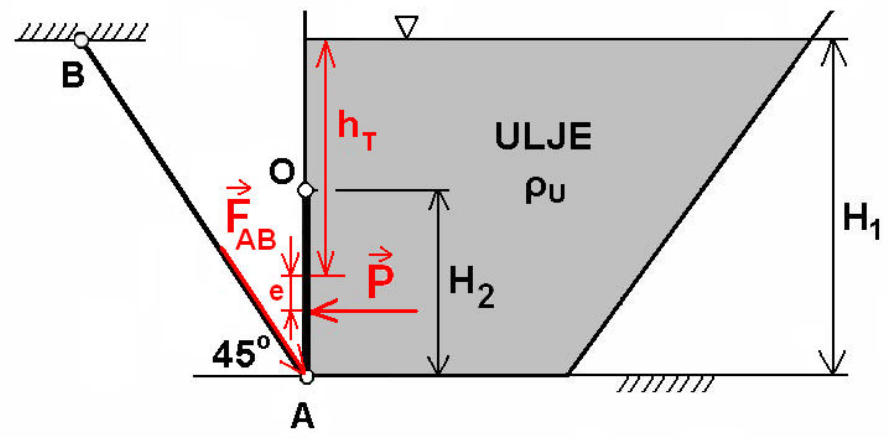
$$P_u = 800 \cdot 9{,}81 \cdot \frac{1}{2} \cdot \sqrt{3},$$

$$P_u = 6796{,}6\ \ N.$$

4. Zadatak

U rezervoaru širine B = 3 m, prikazanom na skici, nalazi se ulje (ρ_u = 800 kg/m^3) do visine H_1 = 4 m. Odrediti:

a) silu koja djeluje na zatvarač visine H_2 = 2 m i širine B = 3 m;
b) vrijednost sile u štapu AB, koji je pod uglom od 45^o u odnosu na horizontalu.



Rješenje:

a)

$$P = \rho_u \cdot g \cdot h_T \cdot A,$$

$$h_T = H_1 - \frac{H_2}{2} = 4 - \frac{2}{2} = 3 \ \ m,$$

$$A = H_2 \cdot B = 2 \cdot 3 = 6 \ \ m^2,$$

$$P = 800 \cdot 9{,}81 \cdot 3 \cdot 6,$$

$$P = 141264 \ \ N.$$

b)

$$\Sigma M_O = 0,$$

$$P \cdot \left(\frac{H_2}{2} + e \right) - F_{AB} \cdot H_2 \cdot \sin 45^o = 0.$$

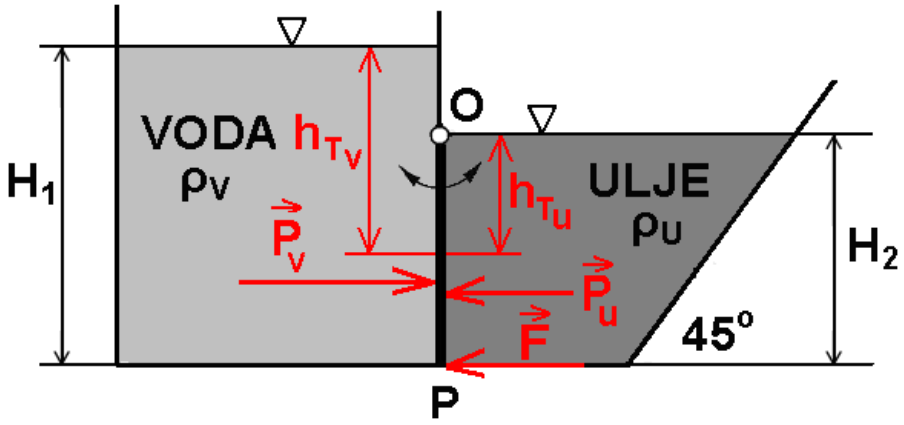
$$e = \frac{I_{XX'}}{h_T \cdot A} = \frac{\frac{B}{12} \cdot H_2^3}{\left(H_1 - \frac{H_2}{2} \right) \cdot H_2 \cdot B} = \frac{\frac{3}{12} \cdot 2^3}{\left(4 - \frac{2}{2} \right) \cdot 2 \cdot 3} = 0{,}11 \ \ m,$$

$$F_{AB} = \frac{P \cdot \left(\frac{H_2}{2} + e \right)}{H_2 \cdot \sin 45^o} = \frac{141264 \cdot \left(\frac{2}{2} + 0{,}11 \right)}{2 \cdot \frac{\sqrt{2}}{2}} = 111207{,}82 \ \ N.$$

5. Zadatak

Zatvarač *OP* između dva spremnika (vidi skicu) može se okretati bez trenja oko ose *O*. Odrediti silu kojom treba djelovati na zatvarač u tačci *P* da bi on bio u ravnoteži u vertikalnom položaju.

Podaci: H_1 = 4 *m*, H_2 = 3 *m*, ρ_V = 1 000 *kg/m*^{3}, ρ_U = 800 *kg/m*^{3}, *B* = 2 *m* (normalno na ravan crteža).



Rješenje:

$$P_v = \rho_v \cdot g \cdot h_{T_v} \cdot A,$$

$$h_{T_v} = H_1 - \frac{H_2}{2} = 4 - \frac{3}{2} = 2{,}5 \ \ m,$$

$$A = H_2 \cdot B = 3 \cdot 2 = 6 \ \ m^2,$$

$$P_v = 1000 \cdot 9{,}81 \cdot 2{,}5 \cdot 6,$$

$$P_v = 147150 \ \ N.$$

$$P_u = \rho_u \cdot g \cdot h_{T_u} \cdot A,$$

$$h_{T_u} = \frac{H_2}{2} = \frac{3}{2} = 1{,}5 \ \ m,$$

$$A = H_2 \cdot B = 3 \cdot 2 = 6 \ \ m^2,$$

$$P_u = 800 \cdot 9{,}81 \cdot 1{,}5 \cdot 6,$$

$$P_u = 70632 \ \ N.$$

$$e_v = \frac{I_{xx'}}{h_{T_v} \cdot A} = \frac{\frac{B}{12} \cdot H_2^3}{\left(H_1 - \frac{H_2}{2}\right) \cdot H_2 \cdot \mathrm{B}} = \frac{\frac{2}{12} \cdot 3^3}{\left(4 - \frac{3}{2}\right) \cdot 3 \cdot 2} = \frac{3}{10} \ \ m,$$

$$e_u = \frac{I_{xx'}}{h_{T_u} \cdot A} = \frac{\frac{B}{12} \cdot H_2^3}{\frac{H_2}{2} \cdot H_2 \cdot \mathrm{B}} = \frac{\frac{2}{12} \cdot 3^3}{\frac{3}{2} \cdot 3 \cdot 2} = \frac{1}{2} \ \ m.$$

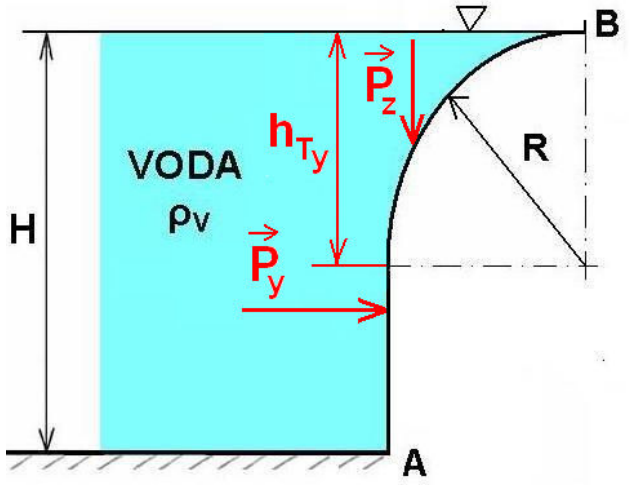
$$\Sigma M_O = 0,$$
$$P_v \cdot x_v - P_u \cdot x_u - F \cdot H_2 = 0.$$

$$x_v = e_v + \frac{H_2}{2} = \frac{3}{10} + \frac{3}{2} = \frac{9}{5} \ \ m,$$
$$x_u = e_u + \frac{H_2}{2} = \frac{1}{2} + \frac{3}{2} = \frac{4}{2} = 2 \ \ m.$$

$$F = \frac{P_v \cdot x_v - P_u \cdot x_u}{H_2} = \frac{147150 \cdot \frac{9}{5} - 70632 \cdot 2}{3} = 41202 \ \ N.$$

6. Zadatak

Odrediti vrijednost horizontalne i vertikalne komponente sile kojom vodeni stub (ρ_v = 1000 kg/m^3) visine H = 4 m djeluje na branu AB širine B = 4 m (normalno na ravan crteža), ako je R = 2 m.



Rješenje:

$$P_y = \rho_v \cdot g \cdot h_{T_y} \cdot A_y,$$

$$h_{T_y} = \frac{H}{2} = \frac{4}{2} = 2 \ \ m,$$

$$A_y = H \cdot B = 4 \cdot 4 = 16 \ \ m^2,$$

$$P_y = 1000 \cdot 9{,}81 \cdot 2 \cdot 16,$$

$$P_y = 313920 \ \ N.$$

$$P_z = \rho_v \cdot g \cdot V,$$

$$V = \left(R^2 - \frac{1}{4} \cdot R^2 \cdot \pi \right) \cdot B,$$

$$V = \left(2^2 - \frac{1}{4} \cdot 2^2 \cdot 3{,}14 \right) \cdot 4,$$

$$V = (4 - 3{,}14) \cdot 4,$$

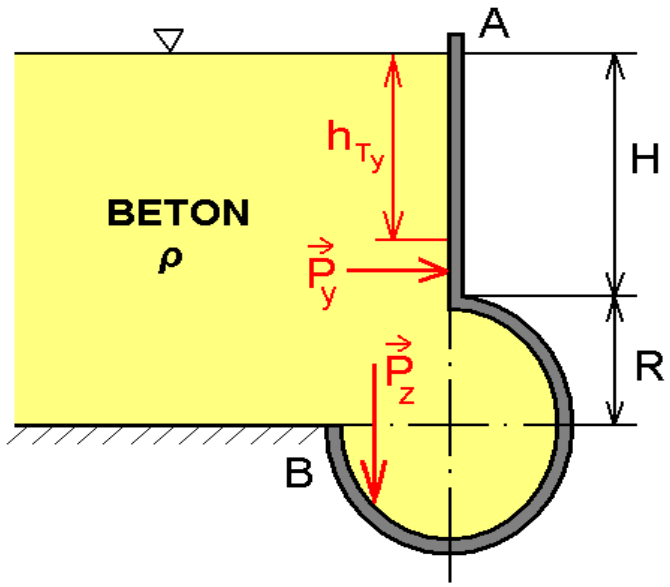
$$V = 3{,}44 \ \ m^3,$$

$$P_z = 1000 \cdot 9{,}81 \cdot 3{,}44,$$

$$P_z = 33746{,}4 \ \ N.$$

7. Zadatak

Odrediti intenzitet horizontalne i vertikalne sile kojom tečni beton (ρ = 2 500 kg/m^3) djeluje na oplatu *AB*, širine *B* = 2 *m* (normalno na ravan crteža), prema skici, ako je *H* = 2 *m*, *R* = 1 *m*.



Rješenje:

$$P_y = \rho \cdot g \cdot h_{T_y} \cdot A_y,$$

$$h_{T_y} = \frac{H+R}{2} = \frac{2+1}{2} = 1{,}5 \ \ m,$$

$$A_y = (H+R) \cdot B = (2+1) \cdot 2 = 6 \ \ m^2,$$

$$P_y = 2500 \cdot 9{,}81 \cdot 1{,}5 \cdot 6,$$

$$P_y = 220725 \ \ N.$$

$$P_z = \rho \cdot g \cdot V,$$

$$V = \left[(H+R) \cdot R + \frac{3}{4} \cdot R^2 \cdot \pi \right] \cdot B,$$

$$V = \left[(2+1) \cdot 1 + \frac{3}{4} \cdot 1^2 \cdot 3{,}14 \right] \cdot 2,$$

$$V = (3 + 2{,}4) \cdot 2,$$

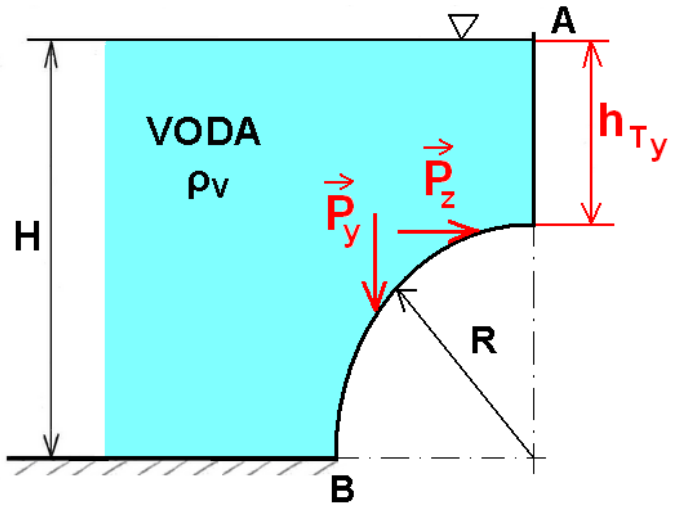
$$V = 10{,}8 \ \ m^3,$$

$$P_z = 2500 \cdot 9{,}81 \cdot 10{,}8,$$

$$P_z = 264870 \ \ N.$$

8. Zadatak

Odrediti vrijednost horizontalne i vertikalne komponente sile kojom vodeni stub (ρ_v = 1000 kg/m^3) visine H = 2 m djeluje na branu AB širine B = 2 m (normalno na ravan crteža), ako je R = 1 m.



Rješenje:

$$P_y = \rho_v \cdot g \cdot h_{T_y} \cdot A_y,$$

$$h_{T_y} = \frac{H}{2} = \frac{2}{2} = 1 \ m,$$

$$A_y = H \cdot B = 2 \cdot 2 = 4 \ m^2,$$

$$P_y = 1000 \cdot 9{,}81 \cdot 1 \cdot 4,$$

$$P_y = 39240 \ N.$$

$$P_z = \rho_v \cdot g \cdot V,$$

$$V = \left[(H - R) \cdot R + R^2 - \frac{1}{4} \cdot R^2 \cdot \pi \right] \cdot B,$$

$$V = \left(H \cdot R - \frac{1}{4} \cdot R^2 \cdot \pi \right) \cdot B,$$

$$V = R \cdot \left(H - \frac{1}{4} \cdot R \cdot \pi \right) \cdot B,$$

$$V = 1 \cdot \left(2 - \frac{1}{4} \cdot 1 \cdot 3{,}14 \right) \cdot 2,$$

$$V = 1 \cdot 1{,}22 \cdot 2,$$

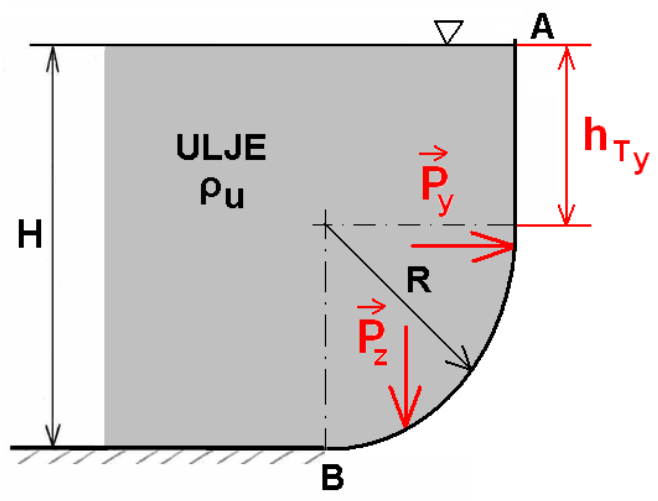
$$V = 2{,}44 \ m^3,$$

$$P_z = 1000 \cdot 9{,}81 \cdot 2{,}44,$$

$$P_z = 23936{,}4 \ N.$$

9. Zadatak

Odrediti vrijednost horizontalne i vertikalne komponente sile kojom uljni stub (ρ_u = 1200 kg/m^3) visine H = 4 m djeluje na branu AB širine B = 4 m (normalno na ravan crteža), ako je R = 2 m.



Rješenje:

$$P_y = \rho_u \cdot g \cdot h_{T_y} \cdot A_y,$$

$$h_{T_y} = \frac{H}{2} = \frac{4}{2} = 2 \ \ m,$$

$$A_y = H \cdot B = 4 \cdot 4 = 16 \ \ m^2,$$

$$P_y = 1200 \cdot 9{,}81 \cdot 2 \cdot 16,$$

$$P_y = 376704 \ \ N.$$

$$P_z = \rho_u \cdot g \cdot V,$$

$$V = \left[(H - R) \cdot R + \frac{1}{4} \cdot R^2 \cdot \pi \right] \cdot B,$$

$$V = \left(H \cdot R - R^2 + \frac{1}{4} \cdot R^2 \cdot \pi \right) \cdot B,$$

$$V = R \cdot \left(H - R + \frac{1}{4} \cdot R \cdot \pi \right) \cdot B,$$

$$V = 2 \cdot \left(4 - 2 + \frac{1}{4} \cdot 2 \cdot 3{,}14 \right) \cdot 4,$$

$$V = 2 \cdot 3{,}57 \cdot 4,$$

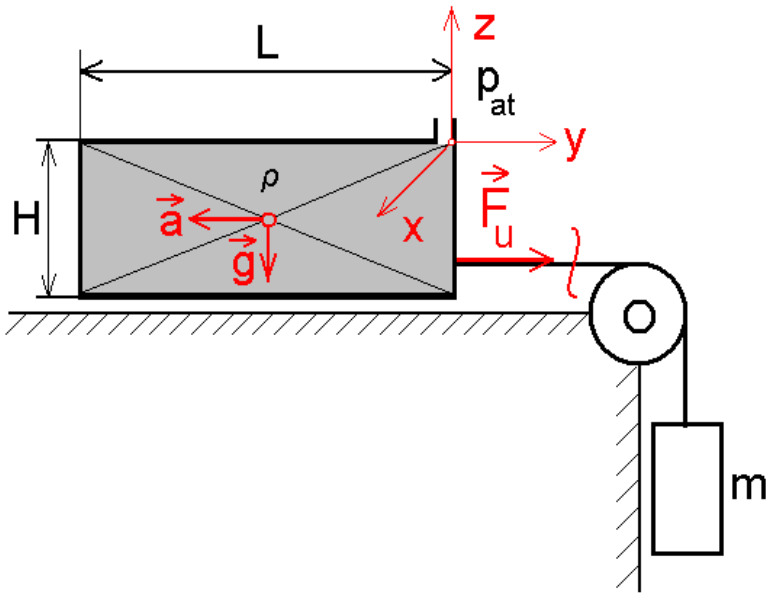
$$V = 28{,}56 \ \ m^3,$$

$$P_z = 1200 \cdot 9{,}81 \cdot 28{,}56,$$

$$P_z = 336208{,}32 \ \ N.$$

10. Zadatak

Pod dejstvom tega mase *m* posuda dužine *L*, visine *H* i širine *B*, napunjena tečnošću gustine *ρ*, kreće se po horizontalnoj ravni konstantnim ubrzanjem. Ako se trenje i masa užeta, kotura i posude zanemare, odrediti jednačinu rasporeda pritiska u posudi.



Rješenje:

$$F_u - \rho \cdot V \cdot a = 0, \quad V = L \cdot H \cdot B,$$
$$mg - ma - F_u = 0,$$

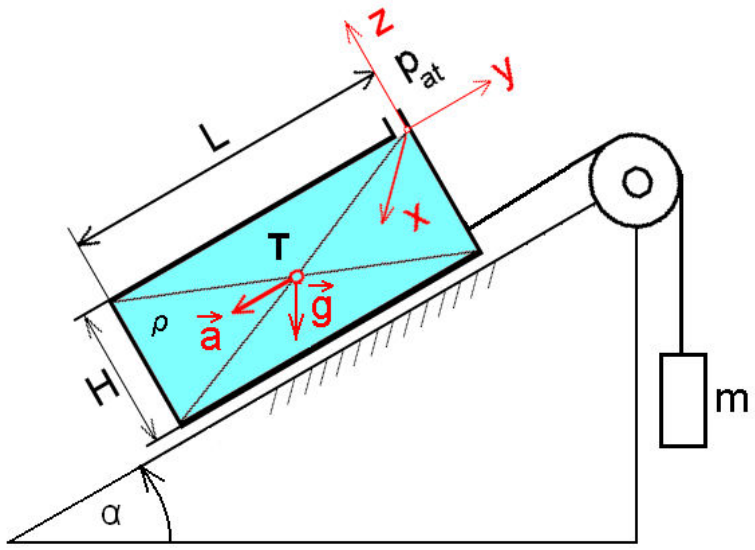
$$\rho \cdot V \cdot a = mg - ma,$$
$$\rho \cdot V \cdot a + ma = mg,$$
$$a \cdot (\rho \cdot V + m) = mg,$$
$$a = \frac{mg}{\rho \cdot V + m}.$$

$$dp = \rho(Xdx + Ydy + Zdz),$$
$$X = 0, \quad Y = -a, \quad Z = -g,$$
$$dp = \rho(-ady - gdz),$$
$$p = \rho(-ay - gz) + C,$$

$$p = 0, \quad y = 0, \quad z = 0 \Rightarrow C = 0,$$
$$p = -\rho(ay + gz),$$
$$p = -\rho\left(\frac{mg}{\rho \cdot V + m} \cdot y + gz\right).$$

11. Zadatak

Pod dejstvom tega mase m posuda dužine L = 2 m, visine H = 1 m i širine B = 2 m, napunjena tečnošću gustine ρ = 1000 kg/m^3, kreće se uz kosu ravan nagiba α = 45° konstantnim ubrzanjem a = 2 m/s^2. Ako se trenje i masa užeta, kotura i posude zanemare, odrediti koliki je pritisak u težištu posude.



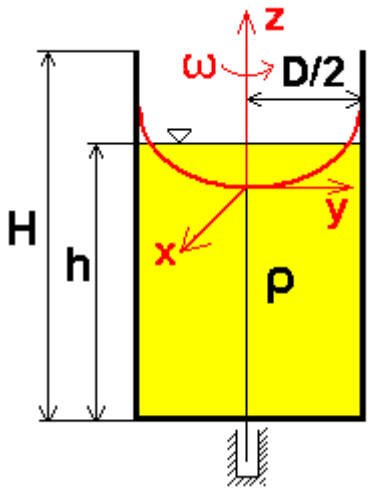
Rješenje:

$$dp = \rho\left(Xdx + Ydy + Zdz\right),$$
$$X = 0,$$
$$Y = -a - g\sin\alpha,$$
$$Z = -g\cos\alpha,$$
$$dp = \rho\left[\left(-a - g\sin\alpha\right)dy - g\cos\alpha dz\right],$$
$$p = \rho\left[\left(-a - g\sin\alpha\right)y - gz\cos\alpha\right] + C,$$
$$p = 0,\ \ y = 0,\ \ z = 0 \Rightarrow C = 0,$$
$$p = \rho\left[\left(-a - g\sin\alpha\right)y - gz\cos\alpha\right].$$

$$p = p_T,\ \ y = -\frac{L}{2},\ \ z = -\frac{H}{2},$$
$$p_T = \rho\left[\left(a + g\sin\alpha\right)\cdot\frac{L}{2} + g\cdot\frac{H}{2}\cos\alpha\right],$$
$$p_T = 1000\cdot\left[\left(2 + 9{,}81\cdot\frac{\sqrt{2}}{2}\right)\cdot\frac{2}{2} + 9{,}81\cdot\frac{1}{2}\cdot\frac{\sqrt{2}}{2}\right],$$
$$p_T = 1000\cdot\left(8{,}92 + 3{,}46\right),$$
$$p_T = 12380\ \ Pa.$$

12. Zadatak

Cilindrični sud prečnika D i visine H napunjen je tečnošću gustine ρ do visine h = 3/4 H i obrće se oko svoje vertikalne ose konstantnom ugaonom brzinom ω. Napisati jednačinu slobodne površine tečnosti i jednačinu za određivanje vrijednosti pritiska tečnosti u tačci koja se nalazi na obrtnoj osi.



Rješenje:

$$dp = \rho(Xdx + Ydy + Zdz),$$

$$X = \omega^2 x,\ \ Y = \omega^2 y,\ \ Z = -g,$$

$$dp = \rho(\omega^2 x dx + \omega^2 y dy - g dz),$$

$$p = \rho\left(\omega^2 \frac{x^2}{2} + \omega^2 \frac{y^2}{2} - gz\right) + C,$$

$$p = 0,\ \ x = 0,\ \ y = 0,\ \ z = 0 \Rightarrow C = 0,$$

$$p = \frac{1}{2}\rho(\omega^2 x^2 + \omega^2 y^2 - 2gz).$$

$$p = 0,$$

$$0 = \frac{1}{2}\rho(\omega^2 x^2 + \omega^2 y^2 - 2gz),$$

$$0 = \frac{1}{2}\rho\omega^2(x^2 + y^2) - \rho g z,$$

$$\rho g z = \frac{1}{2}\rho\omega^2(x^2 + y^2),$$

$$2gz = \omega^2(x^2 + y^2),$$

$$z = \frac{\omega^2}{2g}(x^2 + y^2).$$

$$x = 0,\ \ y = 0,$$

$$p = \frac{1}{2}\rho(\omega^2 \cdot 0^2 + \omega^2 \cdot 0^2 - 2gz),$$

$$p = -\rho g z.$$