UNIVERZITET U ISTOČNOM SARAJEVU
FAKULTET ZA PROIZVODNJU I MENADŽMENT TREBINJE
STUDIJSKI PROGRAM: Industrijsko inženjerstvo za energetiku
PREDMET: Mehanika fluida

RIJEŠENI ZADACI IZ DINAMIKE FLUIDA

Predmetni saradnik:
Mr Radoslav Vučurević

Predmetni nastavnik:
Doc. dr Željko Ratković

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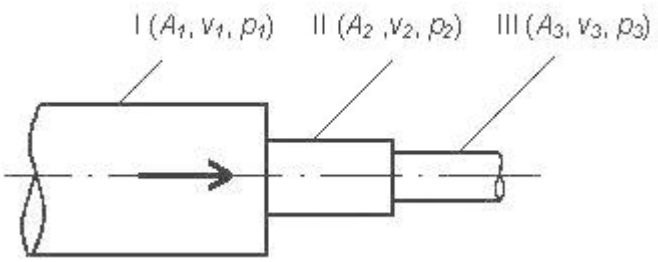
1. Zadatak

Izračunati brzinu fluida u presjeku I cijevi i pritisak u presjeku III cijevi, ako su dati sledeći podaci: ρ = 1000 *kg/m*^{3}, A_1 = 0,2 *m*^{2}, A_2 = 0,1 *m*^{2}, p_{2aps} = 1,6 *bar*, A_3 = 0,05 *m*^{2}, v_3 = 8 *m/s*. Smjer strujanja je dat na skici. Fluid smatrati idealnim i nestišljivim.



Rješenje:

Jednačina kontinuiteta 1-3:

$$\rho \cdot v_1 \cdot A_1 = \rho \cdot v_3 \cdot A_3,$$
$$v_1 = v_3 \cdot \frac{A_3}{A_1},$$
$$v_1 = 8 \cdot \frac{0{,}05}{0{,}2},$$
$$v_1 = 2 \ \ m/s.$$

Jednačina kontinuiteta 2-3:

$$\rho \cdot v_2 \cdot A_2 = \rho \cdot v_3 \cdot A_3,$$
$$v_2 = v_3 \cdot \frac{A_3}{A_2},$$
$$v_2 = 8 \cdot \frac{0{,}05}{0{,}1},$$
$$v_2 = 4 \ \ m/s.$$

Energijska jednačina 2-3:

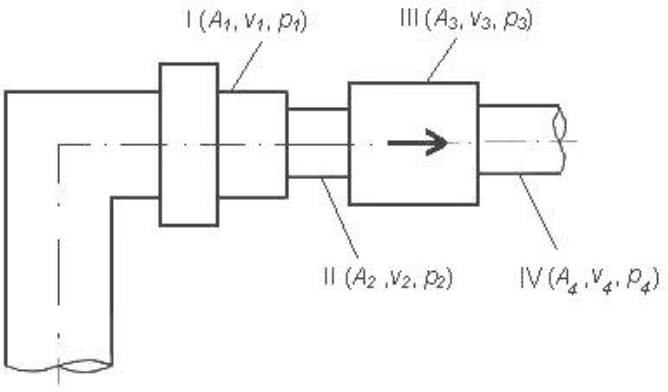
$$\frac{v_2^2}{2g} + \frac{p_2}{\rho g} + z_2 = \frac{v_3^2}{2g} + \frac{p_3}{\rho g} + z_3,$$

$$z_2 = z_3,$$

$$\frac{v_2^2}{2g} + \frac{p_2}{\rho g} = \frac{v_3^2}{2g} + \frac{p_3}{\rho g},$$
$$\frac{p_3}{\rho g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} - \frac{v_3^2}{2g} / \ \cdot \rho g,$$
$$p_3 = p_2 + \rho g \cdot \left(\frac{v_2^2}{2g} - \frac{v_3^2}{2g} \right),$$
$$p_3 = p_2 + \rho \cdot \left(\frac{v_2^2}{2} - \frac{v_3^2}{2} \right),$$
$$p_3 = 1{,}6 \cdot 10^5 + 1000 \cdot \left(\frac{4^2}{2} - \frac{8^2}{2} \right),$$
$$p_3 = 136000 \ \ Pa.$$

2. Zadatak

Izračunati brzinu u presjeku II cijevi, pritisak u presjeku III i silu koja djeluje na suženje između presjeka III i IV. Dati su podaci: ρ = 800 kg/m^3, A_1 = 0,2 m^2, v_1 = 5 m/s, A_2 = 0,1 m^2, p_{2aps} = 1,2 bar, v_3 = 4 m/s, $A_4 = A_2$. Smjer strujanja je dat na skici. Fluid smatrati idealnim i nestišljivim.



Rješenje:

Jednačina kontinuiteta 1-2:

$$\rho \cdot v_1 \cdot A_1 = \rho \cdot v_2 \cdot A_2,$$
$$v_2 = v_1 \cdot \frac{A_1}{A_2},$$
$$v_2 = 5 \cdot \frac{0{,}2}{0{,}1},$$
$$v_2 = 10 \ \ m/s.$$

Energijska jednačina 2-3:

$$\frac{v_2^2}{2g} + \frac{p_2}{\rho g} + z_2 = \frac{v_3^2}{2g} + \frac{p_3}{\rho g} + z_3,$$
$$z_2 = z_3,$$
$$\frac{v_2^2}{2g} + \frac{p_2}{\rho g} = \frac{v_3^2}{2g} + \frac{p_3}{\rho g},$$
$$\frac{p_3}{\rho g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} - \frac{v_3^2}{2g} \ \ / \cdot \rho g,$$
$$p_3 = p_2 + \rho g \cdot \left(\frac{v_2^2}{2g} - \frac{v_3^2}{2g} \right),$$
$$p_3 = p_2 + \rho \cdot \left(\frac{v_2^2}{2} - \frac{v_3^2}{2} \right),$$
$$p_3 = 1{,}2 \cdot 10^5 + 800 \cdot \left(\frac{10^2}{2} - \frac{4^2}{2} \right),$$
$$p_3 = 153600 \ \ Pa.$$

Jednačina kontinuiteta 3-4:

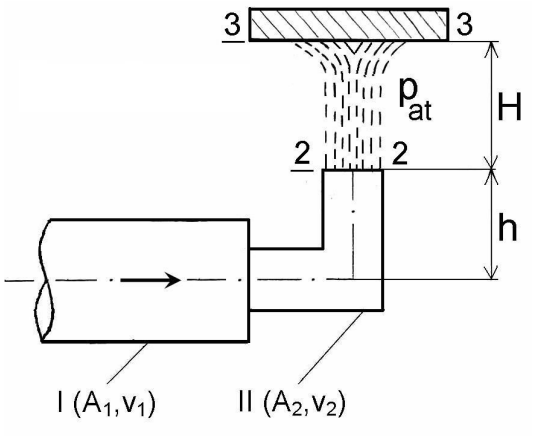
$$\rho \cdot v_3 \cdot A_3 = \rho \cdot v_4 \cdot A_4,$$
$$A_3 = \frac{v_4}{v_3} \cdot A_4,$$
$$A_4 = A_2 = 0{,}1 \ \ m^2,$$
$$v_4 = v_2 = 10 \ \ m/s,$$
$$A_3 = \frac{10}{4} \cdot 0{,}1,$$
$$A_3 = 0{,}25 \ \ m^2.$$

Jednačina količine kretanja:

$$-\rho \cdot v_3^2 \cdot A_3 + \rho \cdot v_4^2 \cdot A_4 = p_3 \cdot A_3 - p_4 \cdot A_4 - F,$$
$$F = \rho \cdot v_3^2 \cdot A_3 - \rho \cdot v_4^2 \cdot A_4 + p_3 \cdot A_3 - p_4 \cdot A_4,$$
$$F = 800 \cdot 4^2 \cdot 0{,}25 - 800 \cdot 10^2 \cdot 0{,}1 + 153600 \cdot 0{,}25 - 120000 \cdot 0{,}1,$$
$$F = 21600 \ \ N.$$

3. Zadatak

Izračunati silu kojom voda (ρ = 1000 kg/m^3) djeluje na glatku ravnu ploču (vidi skicu) ako je A_1=0,2 m^2, v_1= 6 m/s, A_2 = 0,1 m^2, h = 1 m, H = 1 m. Fluid smatrati idealnim i nestišljivim. Smjer strujanja je dat na skici.



Rješenje:

Jednačina kontinuiteta 1-2:

$$\rho \cdot v_1 \cdot A_1 = \rho \cdot v_2 \cdot A_2,$$
$$v_1 \cdot A_1 = v_2 \cdot A_2,$$
$$v_2 = \frac{v_1 \cdot A_1}{A_2},$$
$$v_2 = \frac{6 \cdot 0{,}2}{0{,}1},$$
$$v_2 = 12 \ \ m/s.$$

Energijska jednačina 2-3:

$$\frac{v_2^2}{2g} + \frac{p_2}{\rho g} + z_2 = \frac{v_3^2}{2g} + \frac{p_3}{\rho g} + z_3,$$

$$p_2 = p_3,$$
$$z_3 - z_2 = H,$$

$$\frac{v_2^2}{2g} + z_2 = \frac{v_3^2}{2g} + z_3,$$
$$\frac{v_3^2}{2g} = \frac{v_2^2}{2g} + z_2 - z_3,$$
$$\frac{v_3^2}{2g} = \frac{v_2^2}{2g} - \left(z_3 - z_2\right),$$
$$\frac{v_3^2}{2g} = \frac{v_2^2}{2g} - H \ \ / \cdot 2g,$$
$$v_3^2 = v_2^2 - 2gH,$$
$$v_3 = \sqrt{v_2^2 - 2gH},$$
$$v_3 = \sqrt{12^2 - 2 \cdot 9{,}81 \cdot 1},$$
$$v_3 = 11{,}15 \ \ m/s.$$

Jednačina količine kretanja:

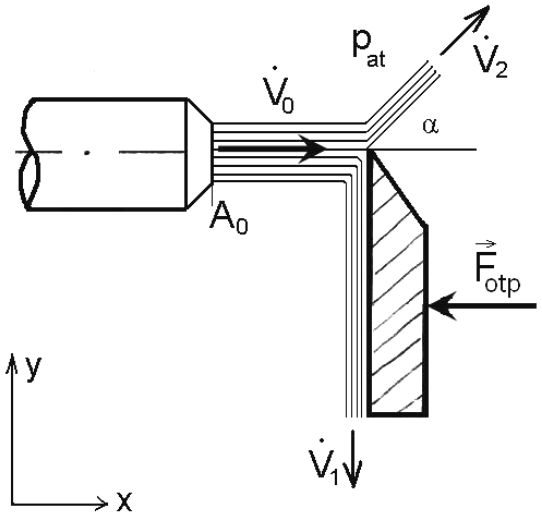
$$-\rho \cdot v_3 \cdot \dot{V} = -F_{otp},$$
$$F_{otp} = \rho \cdot v_3 \cdot \dot{V},$$

$$\dot{V} = v_1 \cdot A_1,$$
$$\dot{V} = 6 \cdot 0{,}2,$$
$$\dot{V} = 1{,}2 \ \ m^3/s,$$

$$F_{otp} = 1000 \cdot 11{,}15 \cdot 1{,}2,$$
$$F_{otp} = 13380 \ \ N.$$

4. Zadatak

Odrediti silu kojom mlaz vode (ρ = 1000 kg/m^3) djeluje na glatku ravnu ploču prikazanu na skici ako je $\dot{V}_0 = 30$ l/s, $\dot{V}_1 = 10$ l/s, A_0= 0,01 m^2. Fluid smatrati idealnim i nestišljivim. Smjer strujanja je dat na skici.



Rješenje:

Jednačina kontinuiteta:

$$\rho \cdot \dot{V}_0 = \rho \cdot \dot{V}_1 + \rho \cdot \dot{V}_2,$$
$$\dot{V}_0 = \dot{V}_1 + \dot{V}_2,$$
$$\dot{V}_2 = \dot{V}_0 - \dot{V}_1,$$
$$\dot{V}_2 = 30 - 10 = 20 \ \ l/s.$$

Energijska jednačina za KZ1:

$$\frac{v_0^2}{2g} + \frac{p_0}{\rho g} + z_0 = \frac{v_1^2}{2g} + \frac{p_1}{\rho g} + z_1,$$

$$p_0 = p_1, \ z_0 = z_1,$$

$$\frac{v_0^2}{2g} = \frac{v_1^2}{2g},$$
$$v_1 = v_0,$$

$$v_0 = \frac{\dot{V}_0}{A_0} = \frac{0{,}03}{0{,}01} = 3 \ \ m/s,$$

$$v_1 = 3 \ \ m/s.$$

Energijska jednačina za KZ2:

$$\frac{v_0^2}{2g}+\frac{p_0}{\rho g}+z_0=\frac{v_2^2}{2g}+\frac{p_2}{\rho g}+z_2,$$

$$p_0=p_2,\ \ z_0=z_2,$$

$$\frac{v_0^2}{2g}=\frac{v_2^2}{2g},$$
$$v_2=v_0,$$
$$v_2=3\ \ m/s.$$

Jednačina količine kretanja za x osu:

$$-v_0\cdot\rho\cdot\dot{V}_0+v_2\cdot\rho\cdot\dot{V}_2\cdot\cos\alpha=-F_{otp}.\ \ (1)$$

Jednačina količine kretanja za y osu:

$$-v_1\cdot\rho\cdot\dot{V}_1+v_2\cdot\rho\cdot\dot{V}_2\cdot\sin\alpha=0,$$
$$v_2\cdot\rho\cdot\dot{V}_2\cdot\sin\alpha=v_1\cdot\rho\cdot\dot{V}_1,$$
$$\sin\alpha=\frac{v_1\cdot\rho\cdot\dot{V}_1}{v_2\cdot\rho\cdot\dot{V}_2},$$

$$v_1=v_2,$$

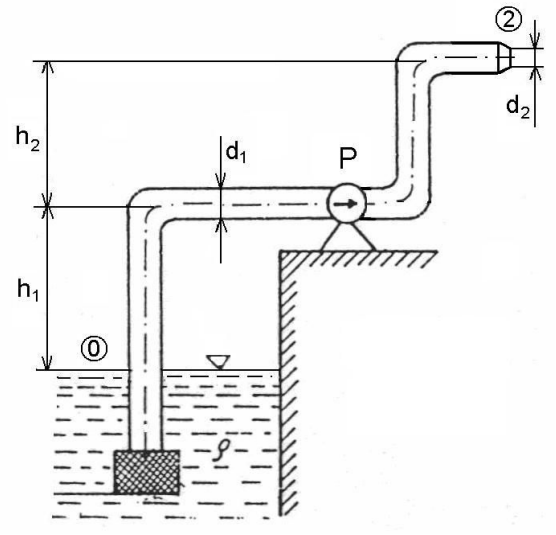
$$\sin\alpha=\frac{\dot{V}_1}{\dot{V}_2}=\frac{10}{20}=\frac{1}{2}\Rightarrow\alpha=30^o.$$

Iz (1):

$$F_{otp}=v_0\cdot\rho\cdot\dot{V}_0-v_2\cdot\rho\cdot\dot{V}_2\cdot\cos\alpha,$$
$$F_{otp}=3\cdot1000\cdot0{,}03-3\cdot1000\cdot0{,}02\cdot\frac{\sqrt{3}}{2},$$
$$F_{otp}=38{,}03\ \ N.$$

5. Zadatak

Za sistem na skici odrediti snagu pumpe. Dato je h_1 = 3 *m*, h_2 = 2 *m*, d_1 = 300 *mm*, d_2 = 200 *mm*, $\dot{V} = 0{,}1\ m^3/s$, ρ=1000 kg/m^3. Fluid smatrati idealnim i nestišljivim.



Rješenje:

Energijaska jednačina 0-2:

$$\frac{v_0^2}{2g}+\frac{p_0}{\rho g}+z_0=\frac{v_2^2}{2g}+\frac{p_2}{\rho g}+z_2+h_t,$$

$$v_0=0,\ \ p_0=p_2,$$

$$\frac{v_2^2}{2g}+z_2-z_0+h_t=0,$$

$$h_t=-h_P,$$
$$z_2-z_0=h_1+h_2,$$

$$\frac{v_2^2}{2g}+h_1+h_2-h_P=0,$$

$$h_P=\frac{v_2^2}{2g}+h_1+h_2,$$

$$v_2=\frac{\dot{V}}{A_2}=\frac{\dot{V}}{\frac{d_2^2\cdot\pi}{4}}=\frac{4\cdot\dot{V}}{d_2^2\cdot\pi}=\frac{4\cdot 0{,}1}{0{,}2^2\cdot 3{,}14}=3{,}17\ \ m/s,$$

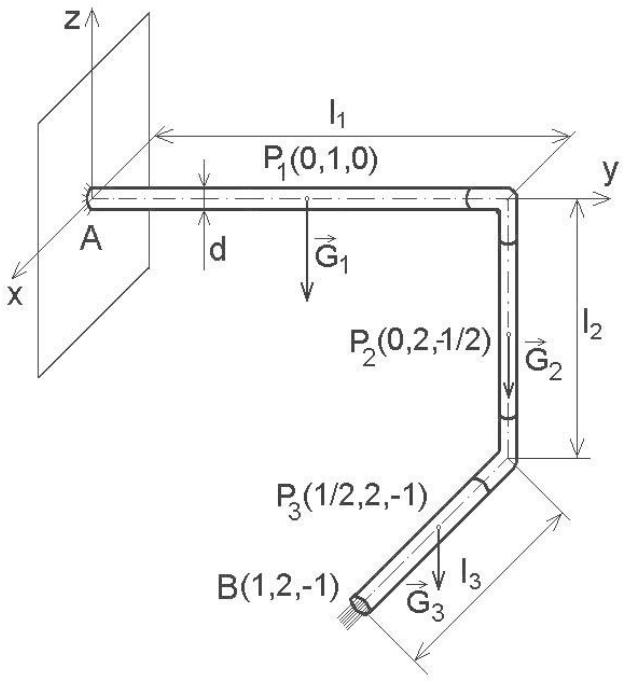
$$h_P=\frac{3{,}17^2}{2\cdot 9{,}81}+3+2,$$
$$h_P=5{,}51\ m,$$

$$N_P=\rho\cdot g\cdot\dot{V}\cdot h_P,$$
$$N_P=1000\cdot 9{,}81\cdot 0{,}1\cdot 5{,}51,$$
$$N_P=5405{,}31\ W.$$

6. Zadatak

Kroz cijev ukliještenu u uklještenju *A* protiče $\dot{V} = 4 \ \ l/s$ vode (ρ = 1000 *kg/m*^{3}). Ako je težina po dužnom metru cijevi $\gamma = 40 \ \ N/m$, izračunati moment uklještenja.

Dato je: d = 50,5 *mm*, l_1 = 2 *m*, l_2 = 1 *m*, l_3 = 1 *m*. Uzeti g = 10 *m/s*^{2}.



Rješenje:

Površina poprečnog presjeka cijevi:

$$A = \frac{d^2 \cdot \pi}{4} = \frac{0{,}0505^2 \cdot 3{,}14}{4} = 0{,}002 \ \ m^2.$$

Brzina u cijevi:

$$v = \frac{\dot{V}}{A} = \frac{0{,}004}{0{,}002} = 2 \ \ m/s.$$

Jednačina momenta količine kretanja:

$$\vec{r}_A x \vec{v}_A \cdot \left(-\rho \dot{V}\right) + \vec{r}_B x \vec{v}_B \cdot \rho \dot{V} = \vec{M}_m + \vec{M}_P, \ \ (1)$$

$$\vec{r}_A x \vec{v}_A = \vec{0},$$
$$\vec{r}_B x \vec{v}_B \cdot \rho \dot{V} = \left(\vec{i} + 2\vec{j} - \vec{k}\right) x 2\vec{i} \cdot 1000 \cdot 0{,}004,$$
$$\vec{r}_B x \vec{v}_B \cdot \rho \dot{V} = \left(\vec{i} + 2\vec{j} - \vec{k}\right) x 8\vec{i},$$
$$\vec{r}_B x \vec{v}_B \cdot \rho \dot{V} = -8\vec{j} - 16\vec{k},$$

$$\vec{M}_m = \vec{r}_1 x \vec{G}_1 + \vec{r}_2 x \vec{G}_2 + \vec{r}_3 x \vec{G}_3,$$

$$\vec{r}_1 x \vec{G}_1 = \vec{j} x \left(\rho g l_1 A + \gamma l_1\right) \cdot \left(-\vec{k}\right),$$
$$\vec{r}_1 x \vec{G}_1 = \vec{j} x \left(1000 \cdot 10 \cdot 2 \cdot 0{,}002 + 40 \cdot 2\right) \cdot \left(-\vec{k}\right),$$
$$\vec{r}_1 x \vec{G}_1 = \vec{j} x \left(-120\vec{k}\right),$$

$$\vec{r}_2 x \vec{G}_2 = \left(2\vec{j} - \frac{1}{2}\vec{k}\right) x (\rho g I_2 A + \gamma I_2) \cdot \left(-\vec{k}\right),$$
$$\vec{r}_2 x \vec{G}_2 = \left(2\vec{j} - \frac{1}{2}\vec{k}\right) x (1000 \cdot 10 \cdot 1 \cdot 0{,}002 + 40 \cdot 1) \cdot \left(-\vec{k}\right),$$
$$\vec{r}_2 x \vec{G}_2 = \left(2\vec{j} - \frac{1}{2}\vec{k}\right) x \left(-60\vec{k}\right),$$

$$\vec{r}_3 x \vec{G}_3 = \left(\frac{1}{2}\vec{i} + 2\vec{j} - \vec{k}\right) x (\rho g I_3 A + \gamma I_3) \cdot \left(-\vec{k}\right),$$
$$\vec{r}_3 x \vec{G}_3 = \left(\frac{1}{2}\vec{i} + 2\vec{j} - \vec{k}\right) x (1000 \cdot 10 \cdot 1 \cdot 0{,}002 + 40 \cdot 1) \cdot \left(-\vec{k}\right),$$
$$\vec{r}_3 x \vec{G}_3 = \left(\frac{1}{2}\vec{i} + 2\vec{j} - \vec{k}\right) x \left(-60\vec{k}\right),$$

$$\vec{M}_m = \vec{j} x \left(-120\vec{k}\right) + \left(2\vec{j} - \frac{1}{2}\vec{k}\right) x \left(-60\vec{k}\right) + \left(\frac{1}{2}\vec{i} + 2\vec{j} - \vec{k}\right) x \left(-60\vec{k}\right),$$
$$\vec{M}_m = -360\vec{i} + 30\vec{j},$$

$$\vec{M}_P = \vec{M}_A.$$

Iz (1):

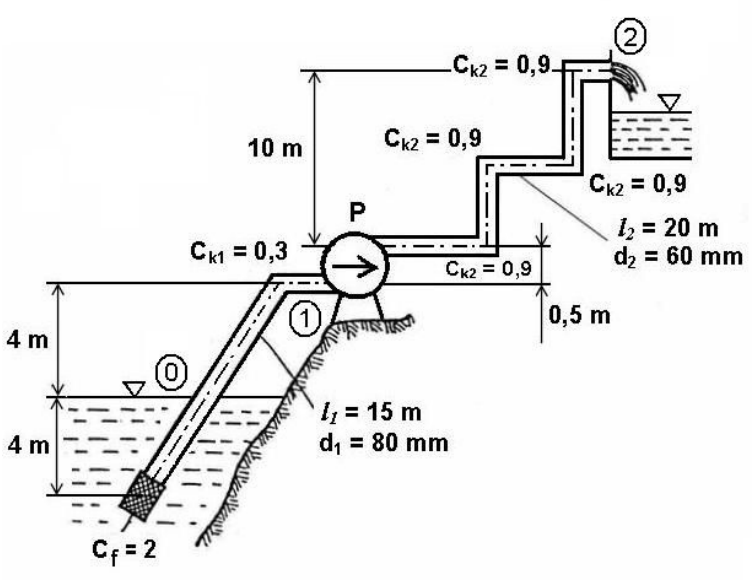
$$\vec{M}_A = \vec{r}_B x \vec{v}_B \cdot \rho \dot{V} - \vec{M}_m,$$
$$\vec{M}_A = -8\vec{j} - 16\vec{k} - \left(-360\vec{i} + 30\vec{j}\right),$$
$$\vec{M}_A = 360\vec{i} - 38\vec{j} - 16\vec{k}.$$

7. Zadatak

Na skici je dat sistem kojim se transportuje tečnost (ρ=1000 *kg/m*^{3}, ν=10^{-5} *m*^{2}*/s*). Odrediti:

a) snagu pumpe, ako je potreban protok 1 *l/s*;

b) pritisak na ulazu u pumpu, ako je atmosferski pritisak 1 bar.



Rješenje:

a)

$$v_{sr1} = \frac{\dot{V}}{\frac{d_1^2 \cdot \pi}{4}} = \frac{4 \cdot \dot{V}}{d_1^2 \cdot \pi} = \frac{4 \cdot 0{,}001}{0{,}08^2 \cdot 3{,}14} = 0{,}2 \ \ m/s,$$

$$R_{e_1} = \frac{v_{sr1} \cdot d_1}{\upsilon} = \frac{0{,}2 \cdot 0{,}08}{10^{-5}} = 1600,$$

$R_{e_1} < 2300$ - strujanje u cijevi prečnika d_1 je laminarno;

$$\lambda_1 = \frac{64}{R_{e_1}} = \frac{64}{1600} = 0{,}04.$$

$$v_{sr2} = \frac{\dot{V}}{\frac{d_2^2 \cdot \pi}{4}} = \frac{4 \cdot \dot{V}}{d_2^2 \cdot \pi} = \frac{4 \cdot 0{,}001}{0{,}06^2 \cdot 3{,}14} = 0{,}36 \ \ m/s,$$

$$R_{e_2} = \frac{v_{sr2} \cdot d_2}{\upsilon} = \frac{0{,}36 \cdot 0{,}06}{10^{-5}} = 2160,$$

$R_{e_2} < 2300$ - strujanje u cijevi prečnika d_2 je laminarno;

$$\lambda_2 = \frac{64}{R_{e_2}} = \frac{64}{2160} = 0{,}03.$$

Energijska jednačina 0-2:

$$\alpha \cdot \frac{v_{sr0}^2}{2g} + \frac{p_0}{\rho g} + z_0 = \alpha \cdot \frac{v_{sr2}^2}{2g} + \frac{p_2}{\rho g} + z_2 + h_t + h_{gub_{0-2}},$$

$$v_{sr0} = 0, \;\; p_0 = p_2 = 0, \;\; \alpha = 2,$$

$$\frac{v_{sr2}^2}{g} + z_2 - z_0 + h_t + h_{gub_{0-2}} = 0,$$

$$h_t = -h_P,$$

$$h_P = \frac{v_{sr2}^2}{g} + z_2 - z_0 + h_{gub_{0-2}},$$

$$z_2 - z_0 = 14{,}5 \;\; m,$$

$$h_{gub_{0-2}} = \frac{v_{sr1}^2}{2g} \cdot \left(\lambda_1 \cdot \frac{l_1}{d_1} + C_f + C_{k_1} \right) + \frac{v_{sr2}^2}{2g} \cdot \left(\lambda_2 \cdot \frac{l_2}{d_2} + 4 \cdot C_{k_2} \right),$$

$$h_{gub_{0-2}} = \frac{0{,}2^2}{2 \cdot 9{,}81} \cdot \left(0{,}04 \cdot \frac{15}{0{,}08} + 2 + 0{,}3 \right) + \frac{0{,}36^2}{2 \cdot 9{,}81} \cdot \left(0{,}03 \cdot \frac{20}{0{,}06} + 4 \cdot 0{,}9 \right),$$

$$h_{gub_{0-2}} = 0{,}12 \;\; m,$$

$$h_P = \frac{0{,}36^2}{9{,}81} + 14{,}5 + 0{,}12,$$

$$h_P = 14{,}63 \;\; m,$$

$$N_P = \rho \cdot g \cdot \dot{V} \cdot h_P,$$

$$N_P = 1000 \cdot 9{,}81 \cdot 0{,}001 \cdot 14{,}63,$$

$$N_P = 143{,}52 \;\; W.$$

b)

Energijska jednačina 0-1:

$$\alpha \cdot \frac{v_{sr0}^2}{2g} + \frac{p_{0aps}}{\rho g} + z_0 = \alpha \cdot \frac{v_{sr1}^2}{2g} + \frac{p_{1aps}}{\rho g} + z_1 + h_{gub_{0-1}},$$

$$v_{sr0} = 0, \quad \alpha = 2,$$

$$\frac{p_{1aps}}{\rho g} = \frac{p_{0aps}}{\rho g} + z_0 - z_1 - \frac{v_{sr1}^2}{g} - h_{gub_{0-1}} \; / \; \cdot \rho g,$$

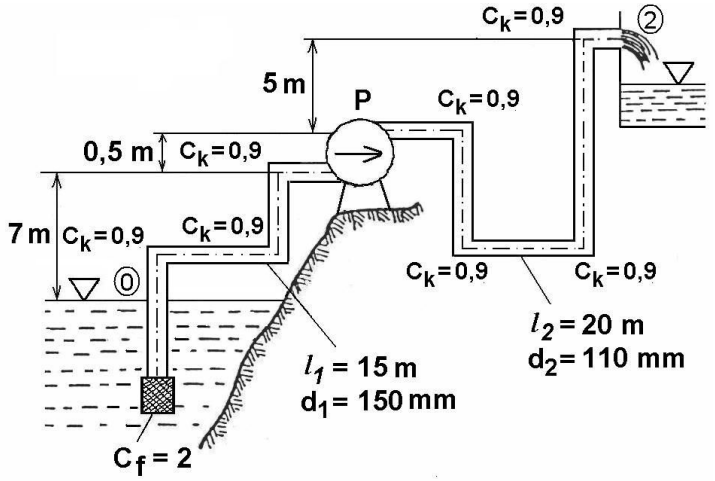
$$p_{1aps} = p_{0aps} + \rho g \cdot \left[-(z_1 - z_0) - \frac{v_{sr1}^2}{g} - h_{gub_{0-1}} \right],$$

$$p_{0aps} = 1 \; bar,$$
$$z_1 - z_0 = 4 \; m,$$
$$h_{gub_{0-1}} = \frac{v_{sr1}^2}{2g} \cdot \left(\lambda_1 \cdot \frac{l_1}{d_1} + C_f + C_{k_1} \right),$$
$$h_{gub_{0-1}} = \frac{0{,}2^2}{2 \cdot 9{,}81} \cdot \left(0{,}04 \cdot \frac{15}{0{,}08} + 2 + 0{,}3 \right),$$
$$h_{gub_{0-1}} = 0{,}02 \; m,$$

$$p_{1aps} = 1 \cdot 10^5 + 1000 \cdot 9{,}81 \cdot \left[-4 - \frac{0{,}2^2}{9{,}81} - 0{,}02 \right],$$
$$p_{1aps} = 60523{,}8 \; Pa.$$

8. Zadatak

Na skici je dat sistem kojim se transportuje tečnost (ρ = 1 000 kg/m^3, ν = 10^{-5} m^2/s). Odrediti snagu pumpe ako je potreban protok 1,5 l/s.



Rješenje:

$$v_{sr1} = \frac{\dot{V}}{\frac{d_1^2 \cdot \pi}{4}} = \frac{4 \cdot \dot{V}}{d_1^2 \cdot \pi} = \frac{4 \cdot 0{,}0015}{0{,}15^2 \cdot 3{,}14} = 0{,}085 \ \ m/s,$$

$$R_{e_1} = \frac{v_{sr1} \cdot d_1}{\upsilon} = \frac{0{,}085 \cdot 0{,}15}{10^{-5}} = 1275,$$

$R_{e_1} < 2300$ - strujanje u cijevi prečnika d_1 je laminarno;

$$\lambda_1 = \frac{64}{R_{e_1}} = \frac{64}{1275} = 0{,}05.$$

$$v_{sr2} = \frac{\dot{V}}{\frac{d_2^2 \cdot \pi}{4}} = \frac{4 \cdot \dot{V}}{d_2^2 \cdot \pi} = \frac{4 \cdot 0{,}0015}{0{,}11^2 \cdot 3{,}14} = 0{,}158 \ \ m/s,$$

$$R_{e_2} = \frac{v_{sr2} \cdot d_2}{\upsilon} = \frac{0{,}158 \cdot 0{,}11}{10^{-5}} = 1730,$$

$R_{e_2} < 2300$ - strujanje u cijevi prečnika d_2 je laminarno;

$$\lambda_2 = \frac{64}{R_{e_2}} = \frac{64}{1730} = 0{,}04.$$

Energijska jednačina 0-2:

$$\alpha \cdot \frac{v_{sr0}^2}{2g} + \frac{p_0}{\rho g} + z_0 = \alpha \cdot \frac{v_{sr2}^2}{2g} + \frac{p_2}{\rho g} + z_2 + h_t + h_{gub_{0-2}},$$

$$v_{sr0} = 0, \;\; p_0 = p_2 = 0, \;\; \alpha = 2,$$

$$\frac{v_{sr2}^2}{g} + z_2 - z_0 + h_t + h_{gub_{0-2}} = 0,$$

$$h_t = -h_P,$$

$$h_P = \frac{v_{sr2}^2}{g} + z_2 - z_0 + h_{gub_{0-2}},$$

$$z_2 - z_0 = 12{,}5 \;\; m,$$

$$h_{gub_{0-2}} = \frac{v_{sr1}^2}{2g} \cdot \left(\lambda_1 \cdot \frac{l_1}{d_1} + C_f + 3 \cdot C_k \right) + \frac{v_{sr2}^2}{2g} \cdot \left(\lambda_2 \cdot \frac{l_2}{d_2} + 4 \cdot C_k \right),$$

$$h_{gub_{0-2}} = \frac{0{,}085^2}{2 \cdot 9{,}81} \cdot \left(0{,}05 \cdot \frac{15}{0{,}15} + 2 + 3 \cdot 0{,}9 \right) + \frac{0{,}158^2}{2 \cdot 9{,}81} \cdot \left(0{,}04 \cdot \frac{20}{0{,}11} + 4 \cdot 0{,}9 \right),$$

$$h_{gub_{0-2}} = 0{,}018 \;\; m,$$

$$h_P = \frac{0{,}158^2}{9{,}81} + 12{,}5 + 0{,}018,$$

$$h_P = 12{,}521 \;\; m,$$

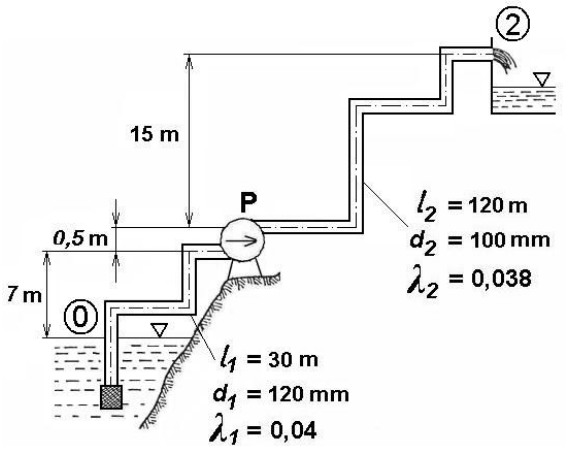
$$N_P = \rho \cdot g \cdot \dot{V} \cdot h_P,$$

$$N_P = 1000 \cdot 9{,}81 \cdot 0{,}0015 \cdot 12{,}521,$$

$$N_P = 184{,}25 \;\; W.$$

9. Zadatak

Na skici je prikazan sistem kojim se transportuje tečnost gustine ρ = 1000 kg/m^3. Odrediti snagu pumpe, ako je strujanje u cjevovodu turbulentno, a protok iznosi 0,01 m^3/s. Lokalne gubitke u cjevovodu zanemariti.



Rješenje:

$$\alpha \cdot \frac{v_{sr0}^2}{2g} + \frac{p_0}{\rho g} + z_0 = \alpha \cdot \frac{v_{sr2}^2}{2g} + \frac{p_2}{\rho g} + z_2 + h_t + h_{gub_{0-2}},$$

$$v_{sr0} = 0, \;\; p_0 = p_2 = 0, \;\; \alpha = 1,$$

$$\frac{v_{sr2}^2}{2g} + z_2 - z_0 + h_t + h_{gub_{0-2}} = 0,$$

$$h_t = -h_P,$$

$$h_P = \frac{v_{sr2}^2}{2g} + z_2 - z_0 + h_{gub_{0-2}},$$

$$z_2 - z_0 = 22{,}5 \;\; m,$$

$$h_{gub_{0-2}} = \frac{v_{sr1}^2}{2g} \cdot \lambda_1 \cdot \frac{l_1}{d_1} + \frac{v_{sr2}^2}{2g} \cdot \lambda_2 \cdot \frac{l_2}{d_2},$$

$$v_{sr1} = \frac{\dot{V}}{A_1} = \frac{\dot{V}}{\frac{d_1^2 \cdot \pi}{4}} = \frac{4 \cdot \dot{V}}{d_1^2 \cdot \pi} = \frac{4 \cdot 0{,}01}{0{,}12^2 \cdot 3{,}14} = 0{,}89 \;\; m/s,$$

$$v_{sr2} = \frac{\dot{V}}{A_2} = \frac{\dot{V}}{\frac{d_2^2 \cdot \pi}{4}} = \frac{4 \cdot \dot{V}}{d_2^2 \cdot \pi} = \frac{4 \cdot 0{,}01}{0{,}1^2 \cdot 3{,}14} = 1{,}27 \;\; m/s,$$

$$h_{gub_{0-2}} = \frac{0{,}89^2}{2 \cdot 9{,}81} \cdot 0{,}04 \cdot \frac{30}{0{,}12} + \frac{1{,}27^2}{2 \cdot 9{,}81} \cdot 0{,}038 \cdot \frac{120}{0{,}1},$$

$$h_{gub_{0-2}} = 4{,}15 \;\; m,$$

$$h_P = \frac{1{,}27^2}{2 \cdot 9{,}81} + 22{,}5 + 4{,}15,$$

$$h_P = 26{,}73 \;\; m,$$

$$N_P = \rho \cdot g \cdot \dot{V} \cdot h_P,$$

$$N_P = 1000 \cdot 9{,}81 \cdot 0{,}01 \cdot 26{,}73,$$

$$N_P = 2622{,}21 \;\; W.$$